

Note: Investigation of a Marx generator imitating a Tesla transformer

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A compact Marx generator was built to mimic a spark-gap Tesla transformer. The generator produced radio-frequency pulses of up to ± 200 kV and ± 15 A with a frequency between 110 and 280 kHz at a repetition rate of 120 Hz. The generator tolerated larger circuit-parameter perturbations than is expected for conventional Tesla transformers. Possible applications include research on the control and laser guiding of spark discharges. *Published by AIP Publishing.* <https://doi.org/10.1063/1.5035286>

Tesla transformers (or Tesla coils) are pulsed-power supplies that generate bursts of radio-frequency alternating current at very high voltages.¹ They are relatively simple, compact, and inexpensive so have been used in a wide range of applications from particle acceleration to insulation testing. Recently, there has been renewed interest in the ability of Tesla transformers to produce long electrical discharges in air because these discharges can be guided by laser filaments.^{2–6} To date, laser-guided discharges from Tesla transformers have achieved a greater enhancement in length than those from other, primarily direct-current supplies,⁷ and additionally can be produced using only a single electrical terminal.³ This makes Tesla transformers and similar supplies attractive for research towards the control of electrical discharges,⁸ their interaction with laser filaments,⁹ the generation of plasma antennas,¹⁰ and the laser guiding of lightning.^{11,12}

Conventionally, Marx generators are used for these applications. Marx generators are pulsed-power supplies that normally generate short pulses of high-voltage direct current instead of radio-frequency current.¹ Compared to Tesla transformers, Marx generators are more straightforward to engineer and their output is more reproducible. Furthermore, Marx generators do not rely on resonant coupling like Tesla transformers, which is sensitive to changes in circuit parameters. This sensitivity is a potential limitation for Tesla transformers in these applications because dynamic changes to their load, such as the evolution of spark discharge, can disrupt this resonant coupling during operation. Marx generators, by contrast, are nearly immune to this sensitivity by design.

Ideally, the best aspects of these two power supplies could be combined in an improved supply. This note demonstrates that Marx generators can be designed to imitate Tesla transformers, combining an output similar to that of a Tesla transformer with the circuit architecture of a Marx generator. Such modified Marx generators are shown to tolerate circuit-parameter changes more than Tesla transformers, making them attractive alternatives to Tesla transformers in the applications mentioned above. While compact,¹³ high-repetition,¹⁴ and inductively loaded⁹ Marx generators and coupled Marx-Tesla circuits¹⁵ exist, this work demonstrates that a modified

Marx generator can mimic a conventional, loosely coupled spark-gap Tesla transformer (SGTT).

Figure 1 shows the modified Marx-generator apparatus, hereafter a Marx coil (MC), which resembles a compact Tesla transformer without a primary coil (omitted because there is no resonant coupling). During operation, it produces repetitive pulses of high voltage at radio frequencies. Like a SGTT, it can be adjusted to produce no, few, or multiple single-ended spark discharges in air depending on the output terminal and power supply configuration. Additionally, it is able to repeatedly breakdown discharge channels from previous pulses, just like SGTTs, as shown by the subtle “banjo” effect of comb-like discharges in Fig. 1(b) and in additional photos and video in the [supplementary material](#).

While the secondary solenoid of a classic SGTT has no components inside, here the solenoid contains a Marx generator as shown in Fig. 1. A plastic U-channel provides a backbone for the Marx generator components that form the circuit in Fig. 2. Inductors are used instead of resistors for fast charging that is enhanced by the solenoid. Additionally, the solenoid provides electric-field grading to reduce stresses on the components inside and suppresses light and sound emission. To prevent flashover, the inductors are immersed in mineral oil and the capacitor leads are insulated with silicone. The MC is charged by alternating current (AC) from a neon sign transformer (NST). NSTs are convenient here and widely used for SGTTs because their current-limited output tolerates short circuiting.

The spark gaps are adjusted so that when the charging voltage is near a maximum the gaps close to erect the Marx generator. Just as in a SGTT, this leads to a pulsed output that repeats at roughly twice the NST AC frequency, or 120 Hz, made of bursts of radio-frequency high voltage. Figure 3 shows a typical waveform captured by measuring the solenoid base current I_s . The burst in Fig. 3(b) is similar to that of a SGTT powered by the same NST, though not identical. Here, the waveform is approximated well by the exponentially decaying oscillation of an RLC circuit and does not have the slowly modulated (“beating”) envelope typical of a SGTT. The red curve is a fit assuming fixed RLC parameters. The solenoid current is nearly spatially uniform, unlike in some SGTTs, allowing the top voltage to be estimated from the base current as $V_t \approx -L_s dI_s/dt$. The inferred peak voltage $|V_t|$ was 201 kV using the variable RLC fit.

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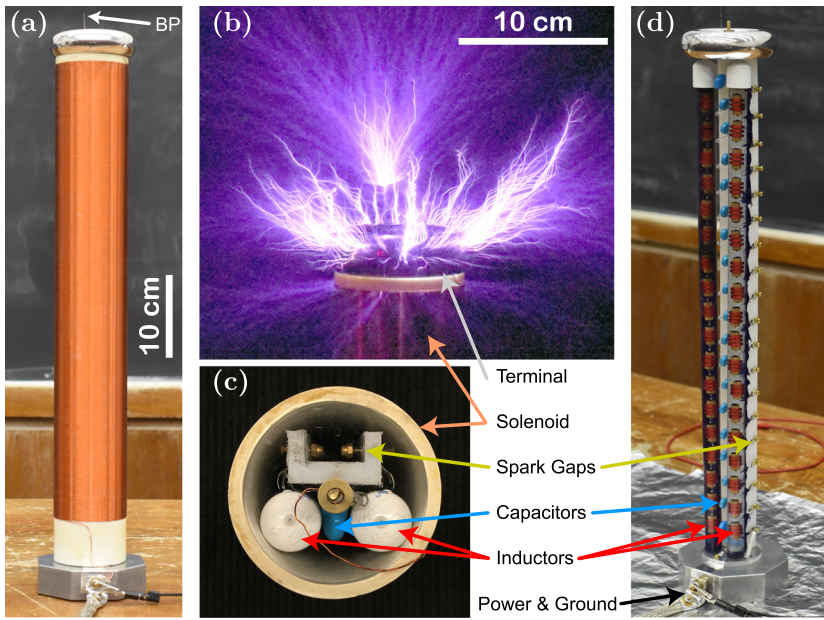


FIG. 1. Marx generator imitating a spark-gap Tesla transformer. (a) Side view showing a metal terminal above a single-layer solenoid inductor and a metal base with power and ground wiring. BP denotes a breakout point to aid spark emission. (b) Spark discharge emitted from the terminal (1/4 s exposure). (c) View beneath the terminal showing the Marx generator inside the solenoid. (d) Side view without the solenoid, showing the Marx generator above a ground plane.

In principle, a Marx generator can be adjusted to mimic a SGT as follows. During operation, a Marx generator charges N stages each with capacitance C_0 in parallel and rapidly rewires the stages in series to produce a pulse. Ideally, the maximum output voltage is N times the charging voltage. By contrast, a SGT charges the capacitance C_p of a primary oscillator circuit and then transfers this energy via resonant coupling to a secondary oscillator circuit with capacitance C_s . From energy conservation, the maximum possible output voltage is $\sqrt{C_p/C_s}$ times the charging voltage.¹ Therefore, for the same charging voltage, choosing $C_0 = C_p/N$ leads to the same energy per pulse and choosing $N = \sqrt{C_p/C_s}$ leads to roughly the same output voltage. To produce an oscillatory output like a SGT, the Marx generator then needs a suitable inductance in parallel with the total erected capacitance, which may come from either an inductive load, the stage impedances, or both.

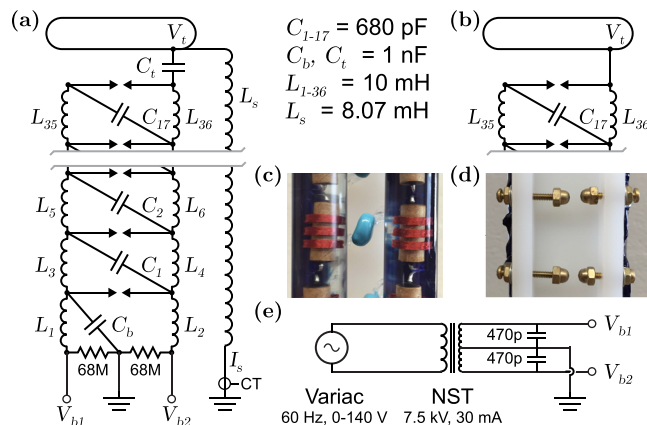


FIG. 2. Circuit details of the Marx coil apparatus. (a) Component wiring for operation with the solenoid, as in Fig. 1(a). CT denotes a current transformer. (b) Modification to operate without the solenoid, as in Fig. 1(d). (c) Inductors (red) and capacitor (blue) for one Marx stage. (d) Spark gaps. (e) Power supply. The Variac was set to 140 V_{rms} for all data shown. Additional details are in the [supplementary material](#).

Choosing the same output frequency leads to a similar output impedance, depending on the spark gap and component losses.

The MC in Fig. 1 was designed by first selecting a power supply and repetition rate common for a compact SGT. Then the charging capacitance was chosen to be near the maximum set by the power supply and rate which limit the energy per burst. The number of stages $N = 18$ was chosen to be

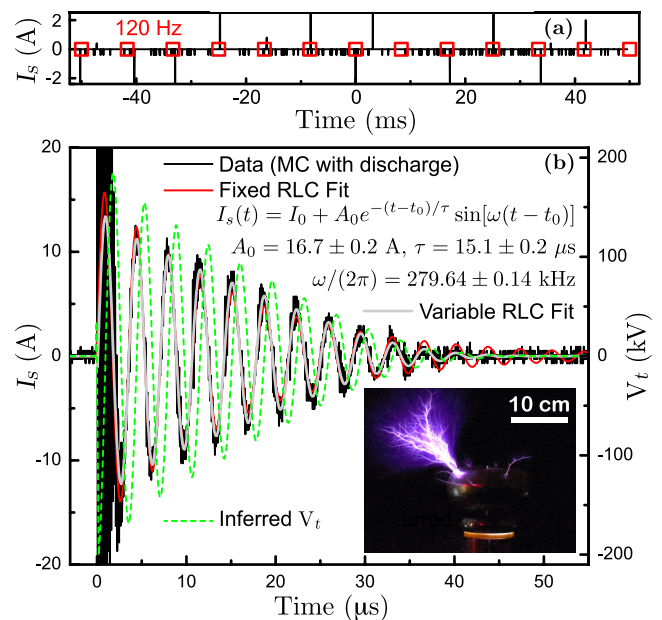


FIG. 3. Output during operation with the solenoid. (a) Like a SGT, the Marx coil produced pulses at a roughly 120 Hz rate, which appear as under-sampled lines in the solenoid current I_s measured by using a current transformer. (b) Current I_s and inferred voltage $V_t \approx -L_s dI_s/dt$ during a pulse. The decaying exponential oscillation of an RLC circuit fits I_s well, but allowing time-varying RLC parameters improves the fit (see the [supplementary material](#)). The inset shows representative spark discharge for these data, which used a larger terminal and a different breakout point than shown in Fig. 1(a).

less than that (~ 35) matching a comparable SGTT to enable the voltage measurement in Fig. 3. (C_b and C_t act as the 18th stage.) The solenoid inductance was then chosen to produce an output oscillation frequency typical of SGTTs. The inductances of the lossy stage inductors were chosen through SPICE simulations to optimize charging speed versus pulse duration.

The sensitivity of Tesla transformers to changes in circuit parameters comes from the resonant coupling that transfers energy between their primary and secondary circuits.^{1,16} Changes that shift the resonant frequency f_0 of either circuit away from their intended values will degrade performance unless the shift δf_0 is roughly within the coupling bandwidth, or approximately f_0/Q using the quality factor Q of the lossy primary. This leads to the rough limit $|\delta f_0/f_0| \lesssim 1/Q$, beyond which the shift impedes energy transfer. Quantitatively, the curve in Fig. 3(b) corresponds to $Q \approx \omega\tau/2 = 13.3 \pm 0.2$, which is similar to a typical SGTT value.¹⁶ Thus for a comparable SGTT, the limit $|\delta f_0/f_0| \lesssim 7.5\%$.

As a result, slightly adjusting the capacitance or inductance of the secondary, for example, typically ruins SGTT performance and either reversing or compensating for this in the primary is needed to restore operation. By contrast, both may be adjusted freely without requiring any other circuit changes to maintain operation with the Marx coil. As a demonstration, Fig. 4 shows the MC operating after the solenoid was removed, as in Figs. 1(d) and 2(b). This reduced the output frequency to about 110 kHz, corresponding to $\delta f/f_0 \approx -61\%$, far outside the rough limit of $\pm 7.5\%$. Additionally, this led to spark discharge off the MC components due to the lack of field grading by the solenoid, as shown in Fig. 4.

In addition to such static changes, SGTT circuit parameters can also change dynamically. In this case, the rough limit

given above holds approximately, although it ignores possibly beneficial effects like rapid adiabatic passage.¹⁷ By contrast, the data in Fig. 4 show a clear frequency ramp from about 140 to 95 kHz, corresponding to $\delta f_0/f_0 \approx -32\%$, highlighting that the MC tolerates dynamic changes. Similar frequency ramps were observed in all data including that in Fig. 3(b), for which it is more subtle. Field-sensitive ceramic stage capacitors are likely responsible for the ramp in Fig. 3(b), and together with stage-inductor saturation are likely responsible for the ramp in Fig. 4.

While the sources of dynamic changes observed here can be removed by replacing components, other sources may be unavoidable. In particular, the development and evolution of transient spark discharge dynamically loads supplies like SGTTs. For example, growing a long leader-like structure effectively loads the supply with ~ 3 pF/m of length.¹⁸ Unfortunately, no reproducible trend was observed that could be attributed to spark discharge, likely because of a larger variability in component effects.

This electrical loading from spark discharge is one potential obstacle to future research with Tesla transformers towards the laser guiding of long sparks because their output capacitance is typically small (~ 20 – 50 pF). Unfortunately, the effects of such discharge loading on Tesla transformers have not been extensively studied (see the [supplementary material](#)).

In summary, a Marx generator was modified to mimic a Tesla transformer, producing similar output and spark discharge. This apparatus tolerated larger changes in its circuit parameters than is expected for Tesla transformers. Thus, such supplies may be attractive alternatives to Tesla transformers in research with the production, control, and laser guiding of spark discharges.

See [supplementary material](#) for additional apparatus, analysis, and spark discharge details (including video).

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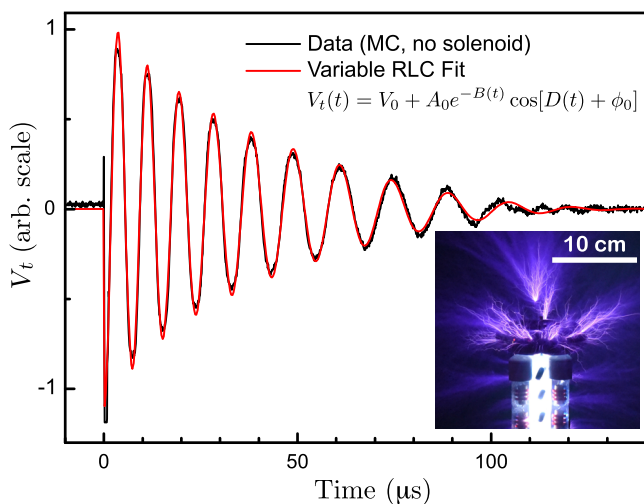


FIG. 4. Output during operation without the solenoid. The voltage V_t oscillates with a frequency that ramps from roughly 140 to 95 kHz, as measured by using an uncalibrated capacitive-pickup probe. Here, the data require time-varying RCL parameters to be fit well (see the [supplementary material](#)), in contrast to Fig. 3(b) that has a more subtle ramp. The inset shows representative spark discharge for this data. Without the solenoid, discharge also occurs off the components.

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