

New Limits on Coupling of Fundamental Constants to Gravity Using ^{87}Sr Optical Lattice Clocks

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The $^1S_0\text{-}^3P_0$ clock transition frequency ν_{Sr} in neutral ^{87}Sr has been measured relative to the Cs standard by three independent laboratories in Boulder, Paris, and Tokyo over the last three years. The agreement on the 1×10^{-15} level makes ν_{Sr} the best agreed-upon optical atomic frequency. We combine periodic variations in the ^{87}Sr clock frequency with $^{199}\text{Hg}^+$ and H-maser data to test local position invariance by obtaining the strongest limits to date on gravitational-coupling coefficients for the fine-structure constant α , electron-proton mass ratio μ , and light quark mass. Furthermore, after $^{199}\text{Hg}^+$, $^{171}\text{Yb}^+$, and H, we add ^{87}Sr as the fourth optical atomic clock species to enhance constraints on yearly drifts of α and μ .

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Frequency is the physical quantity that has been measured with the highest accuracy. While the second is still defined in terms of the radio-frequency hyperfine transition of ^{133}Cs , the higher precision and lower systematic uncertainty achieved in recent years with optical frequency standards promises tests of fundamental physics concepts with increased resolution. For example, some cosmological models imply that fundamental constants and thus atomic frequencies had different values in the early Universe, suggesting that they might still be changing. Records of atomic clock frequencies measured against the Cs standard can be analyzed [1,2] to obtain upper limits on present-day variations of fundamental constants such as the fine-structure constant $\alpha = e^2/(4\pi\epsilon_0\hbar c)$ or the electron-proton mass ratio $\mu = m_e/m_p$ [3–7]. Some unification theories imply violation of local position invariance by predicting coupling of these constants to the ambient gravitational field. Such a dependence could be tested with a deep-space clock mission [8], but would also be observable in the frequency record of earth-bound clocks as Earth's elliptic orbit takes the clock through a varying solar gravitational potential [9]. Annual changes in clock frequencies can thus constrain gravitational coupling of fundamental constants [7,10,11]. Good constraints obtained from such analyses require high confidence in the data and a fast sampling rate. However, a full evaluation of an atomic clock system takes several days so that high-accuracy frequency data are naturally sparse.

Three laboratories have measured the doubly forbidden ^{87}Sr $^1S_0\text{-}^3P_0$ intercombination line at $\nu_{\text{Sr}} \approx$

429 228 004 229 874 Hz with high accuracy over the last three years. These independent laboratories in Boulder (USA), Paris (France), and Tokyo (Japan) agree at the level of 1.7 Hz [12–15]. The agreement between Boulder and Paris is 1×10^{-15} [12–14], approaching the Cs limit, which speaks for the Sr lattice clock system as a candidate for future redefinition of the SI second and makes ν_{Sr} the best agreed-upon optical clock frequency. In this Letter, we analyze the international Sr frequency record for long-term variations and combine our results with data from other atomic clock species to obtain the strongest limits to date on coupling of fundamental constants to gravity. In addition, our data contribute a high-accuracy measurement of an optical atomic clock species, which itself has low sensitivity to variation in fundamental constants, to the search for drifts of fundamental constants, improving confidence in the null result at the current level of accuracy.

In a strontium lattice clock, neutral fermionic ^{87}Sr atoms are trapped at the antinodes of a vertical one-dimensional optical lattice at the Stark-cancellation wavelength, creating an ensemble of nearly identical quantum absorbers at μK temperatures. The $^1S_0\text{-}^3P_0$ clock transition [16] is interrogated with a highly frequency-stabilized 698 nm spectroscopy laser in the resolved sideband limit and the Lamb-Dicke regime [12–15,17,18]. Using individual magnetic sublevels, spectra with quality factors of $>2 \times 10^{14}$ have been recovered [19], as shown in Fig. 1(a). This high-resolution spectroscopy afforded by the optical lattice allows measurement of the clock frequency with high accuracy and evaluation of systematic uncertainties at 1

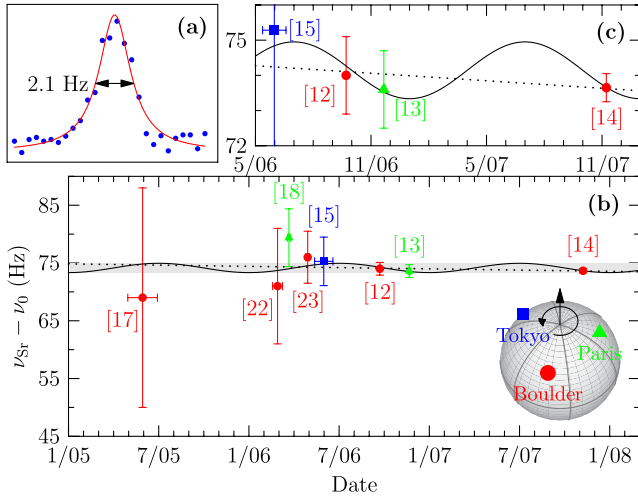


FIG. 1 (color online). (a) Spectrum of the $^{87}\text{Sr } 1S_0-3P_0$ clock transition with quality factor 2×10^{14} . (b) Measurements of the clock transition from JILA (circle), SYRTE (triangle), and U. Tokyo (square) over the last 3 years. Frequency data are shown relative to $\nu_0 = 429\,228\,004\,229\,800$ Hz. Weighted linear (dotted line) and sinusoidal (solid line) fits determine a yearly drift rate and an amplitude of annual variation. (c) Zoom into the four most recent measurements, showing agreement within 1.7 Hz and determining both drift and annual variation.

part in 10^{16} , limited by blackbody and residual density effects [20]. Spectroscopic information from the atomic sample is used to steer the laser to match the clock transition frequency, which is then measured relative to the Cs standard using an octave-spanning optical frequency comb [21].

In combination with data from other optical atomic clock species, variations in the measured Sr clock frequency can constrain variation of fundamental constants. It is necessary to analyze a diverse selection of atomic species to rule out species-dependent systematic effects and test the broad predictions of the underlying relativistic theory. We will introduce the formalism required to constrain the coupling to gravity by first analyzing the global frequency record for linear drifts in α and μ .

Figure 1(b) displays Sr clock frequency measurements since 2005. The frequency uncertainties are based on values from Refs. [12–15,17,18,22,23]. The date error bar indicates the time interval over which each measurement took place. A weighted linear fit (dotted line) results in a frequency drift of $(-1.0 \pm 1.8) \times 10^{-15}/\text{yr}$, mostly determined by the difference between the last three high-accuracy measurements [12–14]. This yearly drift can be related to a drift of fundamental constants via relativistic sensitivity constants K_{rel} . Values for various clock transitions of interest have been calculated in Refs. [24,25], and the fractional variation of an optical transition frequency (in atomic units) can be written as

$$\frac{\delta\nu_{\text{opt}}}{\nu_{\text{opt}}} = K_{\text{rel}}^{\text{opt}} \frac{\delta\alpha}{\alpha}. \quad (1)$$

The Cs standard operates on a hyperfine transition, which is also sensitive to variations in μ . For a hyperfine transition, the above equation is modified to

$$\frac{\delta\nu_{\text{hfs}}}{\nu_{\text{hfs}}} = (K_{\text{rel}}^{\text{hfs}} + 2) \frac{\delta\alpha}{\alpha} + \frac{\delta\mu}{\mu}. \quad (2)$$

Here, the change in μ arises from variations in the nuclear magnetic moment of the Cs atom [25]. The following drift analysis will focus on optical clocks measured against Cs, since inclusion of hyperfine clock data from Rb/Cs [5,26] does not change the results significantly.

The overall fractional frequency variation x_j of an optical clock species j compared to Cs can be related to variation of α and μ as

$$\begin{aligned} x_j &\equiv \frac{\delta(\nu_j/\nu_{\text{Cs}})}{\nu_j/\nu_{\text{Cs}}} = (K_{\text{rel}}^j - K_{\text{rel}}^{\text{Cs}} - 2) \frac{\delta\alpha}{\alpha} - \frac{\delta\mu}{\mu} \\ &\equiv -c_{\alpha}^j \frac{\delta\alpha}{\alpha} - \frac{\delta\mu}{\mu}. \end{aligned} \quad (3)$$

For ^{87}Sr , in particular, $-c_{\alpha}^{\text{Sr}} = 0.06 - 0.83 - 2 = -2.77$ [24]. The ^{87}Sr sensitivity is about 50 times lower than that of Cs, so that our measurements are a clean test of the Cs frequency variation. This allows Sr clocks to serve a similar role as H in removing the Cs contribution from other optical clock experiments or to act as an anchor in direct optical comparisons [3].

Other optical clock species with different sensitivity constants have also been analyzed for frequency drifts. Each species becomes susceptible to variations in both α and μ by referencing to Cs. Figure 2 shows current optical frequency drift rates from Sr, Hg^+ [7], Yb^+ [6], and H [3]. Linear regression [27] limits drift rates to

$$\begin{aligned} \delta\alpha/\alpha &= (-3.3 \pm 3.0) \times 10^{-16}/\text{yr}, \\ \delta\mu/\mu &= (1.6 \pm 1.7) \times 10^{-15}/\text{yr}, \end{aligned} \quad (4)$$

decreasing the H-Yb $^+$ -Hg $^+$ [3,6,7] error bars [28] by $\sim 15\%$ and confirming the null result at the current level of accuracy by adding high-accuracy data from a very insensitive species such as Sr to Fig. 2. We note that another limit on $\delta\alpha/\alpha$ independent of other fundamental constants (using microwave transitions in atomic Dy) has recently been reported as $(-2.7 \pm 2.6) \times 10^{-15}/\text{yr}$ [29].

We will now generalize the formalism used for the analysis of linear drifts to constrain coupling to the gravitational potential U and search for periodic variations in the global frequency record. The dominant contribution to changes in the ambient gravitational potential is due to the ellipticity of Earth's orbit around the Sun. Suppose that the variation of a fundamental constant η is related to the change in gravitational potential via a dimensionless coupling constant k_{η} [9]:

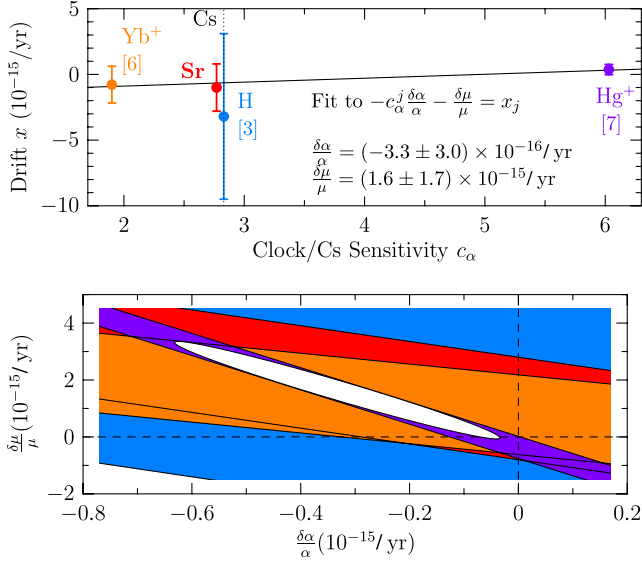


FIG. 2 (color online). The upper panel shows fractional frequency drifts for $^{171}\text{Yb}^+$, ^{87}Sr , H 1S-2S, and $^{199}\text{Hg}^+$ versus their sensitivity to α variation relative to Cs. Sensitivity due to Cs is indicated as a dotted vertical line. A linear fit (solid line) determines yearly drift rates $\delta\alpha/\alpha$ and $\delta\mu/\mu$. The drift rate constraints from each species are shown in the lower panel as shaded bars. The fit determines a confidence ellipse (white) [3,6,7] with projections equal to the parameters' $1\text{-}\sigma$ uncertainties.

$$\frac{\delta\eta}{\eta} \equiv k_\eta \frac{\Delta U(t)}{c^2}, \quad (5)$$

where $\Delta U(t) = U(t) - U_0$ is the variation in the gravitational potential versus the mean solar potential on Earth U_0 , and c is the speed of light.

The variation in solar gravitational potential can then be estimated from Earth's equations of motion (see Fig. 3). Since Earth's orbit is nearly circular, we expand the solar gravitational potential $U(t) = -Gm_\odot/r(t)$, with gravita-

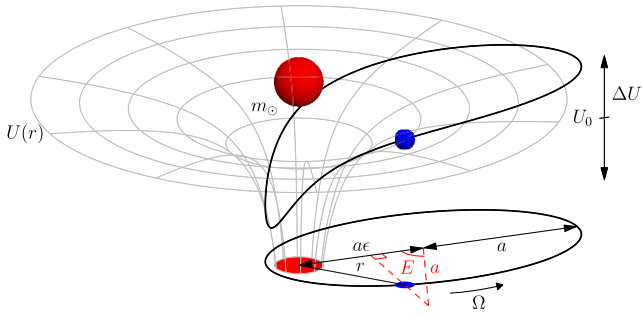


FIG. 3 (color online). Earth orbiting around Sun (mass m_\odot) in gravitational potential U on an orbit with semimajor axis a , eccentricity ϵ (exaggerated to show geometry), and angular velocity Ω . Earth is at radial distance r from the Sun. The eccentric anomaly E is the angle between the major axis and the orthogonal projection of Earth's position onto a circle of radius a .

tional constant G , Sun mass m_\odot , and radial distance Earth-Sun $r(t)$, in the orbit's ellipticity $\epsilon \approx 0.0167$. Kepler's equation [30] relates the eccentric anomaly $E \equiv \arccos[(1 - r/a)/\epsilon]$ (with semimajor axis $a \approx 1$ au) to the orbit's elapsed phase since perihelion:

$$\Omega t = E - \epsilon \sin E, \quad (6)$$

where $\Omega \approx \sqrt{Gm_\odot/a^3} \approx 2 \times 10^{-7} \text{ s}^{-1}$ is Earth's angular velocity from Kepler's third law. Kepler's equation has a solution given by a power series in the ellipticity as $E = \Omega t + \mathcal{O}(\epsilon)$, which can be used to expand $1/r$ and thus ΔU to first order in ϵ :

$$\Delta U(t) = -\frac{Gm_\odot}{a} \epsilon \cos \Omega t, \quad (7)$$

with a dimensionless peak-to-peak amplitude $u \equiv 2Gm_\odot/(ac^2) \approx 3.3 \times 10^{-10}$. Thus, the ^{87}Sr fractional frequency variation due to gravitational coupling is

$$x_{\text{Sr}}(t) = [2.77k_\alpha + k_\mu] \frac{Gm_\odot}{ac^2} \epsilon \cos \Omega t, \quad (8)$$

with amplitude containing k_α and k_μ as the only free parameters. Fitting Eq. (8) to the combined Sr frequency record in Fig. 1(b) gives an annual variation with amplitude $y_{\text{Sr}} = (-1.9 \pm 3.0) \times 10^{-15}$, which constrains $2.77k_\alpha + k_\mu$ by division through u .

Other atomic clock species that have been tested for gravitational coupling are $^{199}\text{Hg}^+$ [7] and the H maser [11]. H masers are also sensitive to variations in the light quark mass [25], adding a third coupling constant k_q . Although the maser operates on a hyperfine transition, the H atom is well understood, permitting the use of H-maser data with optical clocks to constrain k_q . Using sensitivity coefficients from Refs. [24,25], each atomic

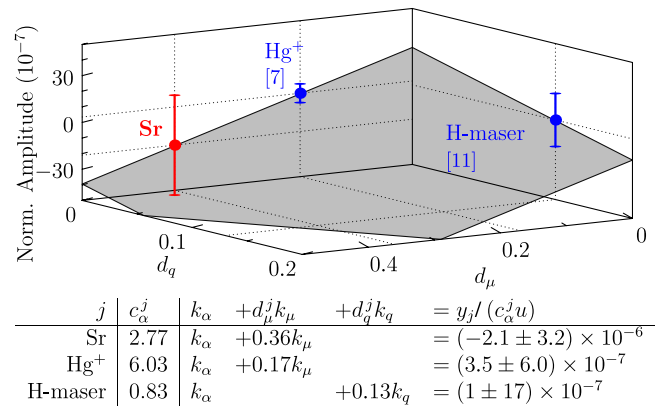


FIG. 4 (color online). A fit to linear constraints on gravitational-coupling constants k_α , k_μ , and k_q from three species determines a plane. Its value at $d_\mu = 0$, $d_q = 0$ is k_α ; its gradient along the d_μ (d_q) axis is k_μ (k_q). The table shows sensitivity constants and constraints for ^{87}Sr , $^{199}\text{Hg}^+$, and the H maser.

clock species j contributes a constraint of the general form [9]

$$c_\alpha^j k_\alpha + c_\mu^j k_\mu + c_q^j k_q = y_j/u. \quad (9)$$

Division by c_α^j gives this equation the form of a linear function in two variables, $d_\mu^j \equiv c_\mu^j/c_\alpha^j$ and $d_q^j \equiv c_q^j/c_\alpha^j$.

In Fig. 4, each species' constraint is interpreted as a measurement of this linear function in the numerical coefficients. The constraint for Hg^+ is corrected for a sign error in applying Eq. 2 of Ref. [7] in the subsequent paragraph. The sign of the constraint for the H maser derives from the averaged fit in Fig. 3 of Ref. [11]. A linear fit gives

$$\begin{aligned} k_\alpha &= (2.5 \pm 3.1) \times 10^{-6}, \\ k_\mu &= (-1.3 \pm 1.7) \times 10^{-5}, \\ k_q &= (-1.9 \pm 2.7) \times 10^{-5}. \end{aligned} \quad (10)$$

Because of the orthogonal dependence on k_q , the maser data only pivots the plane in Fig. 4 around the Hg^+ -Sr line, but its value and error bar influence neither the value nor the error bar of k_α and k_μ . The values agree well with zero and we conclude that there is no coupling of α , μ , and the light quark mass to the gravitational potential at the current level of accuracy. We note that the coupling constant k_α has recently been measured independently in atomic Dy, resulting in $k_\alpha = (-8.7 \pm 6.6) \times 10^{-6}$ [31], limited by systematic effects. While optical clocks are not as sensitive to variations in constants as Dy, systematic effects have been characterized at much higher levels [20].

The unprecedented level of agreement between three international labs on an optical clock frequency allowed precise analysis of the Sr clock data for long-term frequency variations. We have presented the best limits to date on coupling of fundamental constants to the gravitational potential. In addition, by adding a high-accuracy measurement of a low-sensitivity species to the analysis of drifts of fundamental constants, we have increased confidence in the zero drift result for the modern epoch.

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